# Dependence of Electromagnetic Form Factors of Hadrons on Light Cone Frames

H. J. Weber

Institute of Nuclear and Particle Physics, Department of Physics, University of Virginia, Charlottesville, Virginia 22901

Xiaoming Xu

Institute of Nuclear and Particle Physics, Department of Physics, University of Virginia, Charlottesville, Virginia 22901, and Shanghai Institute of Nuclear Research, Chinese Academy of Sciences, Shanghai 201800, China

**Abstract**

A constituent quark model is developed for an arbitrary light-cone direction so that the light-front time is $x^+_{LF} = \omega \cdot x$ with a constant lightlike four-vector $\omega$. Form factors are obtained from free one-body electromagnetic current matrix elements. They are found to be $\omega$-independent for spin-0 mesons, nucleons and the $\Lambda$ hyperon, while there is $\omega$-dependence for spin-1 systems like the deuteron.

## I. Introduction

The light-front formalism of relativistic quantum mechanics has the attractive feature that wave functions may be boosted kinematically, i.e. independent of interactions. Since form factors necessarily involve boosted wave functions, this aspect motivates many recent calculations in the light-front form of relativistic few-body physics [1-8]. A basis of three-quark wave functions for baryons has been constructed in light cone versions of the constituent quark model (LCQM) to study the static properties of the nucleon [1,2] and hyperons [3] and electromagnetic transition form factors for $N \to N^*$ [2, 4] and $N \to \Delta$ processes[2,5]. The three-quark wave function for a nucleon is the product of a totally symmetric momentum wave function and a nonstatic

spin wave function which is an eigenfunction of the total angular momentum (squared) and its projection on the light cone axis. The spin wave function can be represented as a linear combination of products of matrix elements between valence quarks [1,6] coupled by appropriate $\gamma$ (and isospin)-matrices to the spin and isospin of the nucleon. Other nonstatic hadron wave functions are constructed similarly. Direct coupling of spinors by Clebsch-Gordan coefficients leads to equivalent spin-flavor wave functions [2]. Instant spinors are transformed into light-front helicity eigenstates by Melosh transformations [9] which include the kinematic quark boosts. These nonstatic spin invariants resemble the Ioffe currents for the nucleon that are widely used in QCD sum rule techniques [10].

In relativistic quantum mechanics the state vectors of a many-body system transform as a unitary representation of the proper Poincaré group spanned by ten generators, the four-momentum $P_\mu$ and the antisymmetric angular momentum tensor $J_{\mu\nu}$ of the Lorentz group, which satisy the well known commutation relations of the Poincaré algebra. Each form of relativistic quantum mechanics is characterized by initial conditions on a hypersurface of Minkowski space and a subgroup of kinematic (i.e. interaction independent) generators in the stability group that map this hypersurface onto itself [11]. The standard instant form uses $t = constant$, where $t$ is the time, while the front form is based on the null plane $ct + z = 0$ [12]. The noninteracting front form (or light cone) boosts form a subgroup of the null plane's stability group that acts transitively in momentum space. As a result the system's total and internal momentum variables separate, whereas in the instant form no such separation is possible. Thus, front form models are best developed in momentum space. In the instant form the three-momentum $\vec{P}$ and angular momentum $\vec{J}$ are kinematic and act transitively in coordinate space, while the three boost generators (of rotationless Lorentz transformations along the spatial axes) are interaction dependent so that, when the strong interaction is present, one simply cannot use free boosts.

The light-front form is obtained from the instant form in the infinite momentum limit, and this amounts to the well-known change of momentum variables ($p^+ = p^0 + p^z, p^- = p^0 - p^z$) on the light cone [13]. Since in front form both transverse rotation generators become interaction dependent, rotation invariance is more difficult to implement. In essence, light front models include kinematic boosts at the expense of interaction dependent angular momentum operators.

Light front models obtained from quantum field theories usually involve interacting spin operators. There is a unitary (but, as a rule, interaction dependent) transformation to a representation where spins are free and state vectors are specified on a null plane $\omega \cdot x = 0$ with a lightlike (i.e. $\omega^2 = 0$) four-vector $\omega^\mu$ [14,15,16]. If a theory is Lorentz covariant its form factors are expected to be independent of $\omega$. In this case the electromagnetic current operator in general has many-body contributions [17]. When it is taken as a sum of free one-body currents, as in the impulse approximation for example, Lorentz covariance may be violated which then shows up as explicit $\omega$-dependence in form factors. In view of this situation it is worthwhile to test such frame dependence of form factors. The conventional choice of the light cone direction is the z-axis and the light cone time is $x^+ = t + z$. Here we use instead an arbitrary, but constant, lightlike four-vector $\omega^\mu$ to test form factors of the constituent quark model in light front form (LCQM [1-8]) for their frame dependence.

In front form all form factors can be extracted from matrix elements of $J^+$, the so-called 'good' current component [18]. When Lorentz covariance and current conservation are imposed on the electromagnetic current of a few-body system, current matrix elements can be parametrized by a minimal number of Lorentz invariant form factors or multipole moments that are characteristic of the composite system and its quantum numbers. Usually there are more $J^+$ helicity matrix elements than form factors so that several nontrivial constraints must be satisfied. The deuteron is a relevant example for spin 1. Its electromagnetic properties are characterized by the charge, magnetic and quadrupole form factors. Since there are four helicity matrix elements, viz. $< \lambda' = 1 \mid J^+ \mid \lambda = 1 >, < \lambda' = 1 \mid J^+ \mid \lambda = -1 >$, $< \lambda' = 1 \mid J^+ \mid \lambda = 0 >$ and $< \lambda' = 0 \mid J^+ \mid \lambda = 0 >$, there is one angular condition relating them. Often the longitudinal $< \lambda' = 0 \mid J^+ \mid \lambda = 0 >$ matrix element is not used to extract the three form factors because the helicity 0 states are not dominated by two-nucleon states [19]. The angular condition is then also ignored; and there is $\omega$-dependence in the deuteron case [14,15]. The electromagnetic $N \to \Delta$ transition is another relevant case. For the rho-meson with J=1 such an angular condition has been analyzed [20], but the J=0 and 1/2 cases are not considered as there is no angular condition. When Lorentz invariance is violated, additional $\omega$-dependent current components arise along with their form factors which are needed to fully parametrize the current matrix elements that now obey fewer constraints.

When Lorentz invariance is restored by including two-body currents, these spurious currents are cancelled and the remaining physical form factors are modified as well. In other words, $\omega$-independence of form factors is just one step toward Lorentz invariance but, by itself, it does not guarantee correct form factors yet. In the case of $\omega$-dependence the unphysical currents can at least be removed along with inconsistencies between current matrix elements of different helicities.

Since the electromagnetic current matrix elements are calculated with the free current in quark models, the resulting form factors of hadrons may be frame (or $\omega$-) dependent. In this paper we investigate this $\omega$-dependence for spin-0 mesons and spin-$\frac{1}{2}$ baryons in a constituent quark model with boosts [1-8]. In Section II we set up light front dynamics on the initial plane $\omega \cdot x = 0$ by first defining the light-front time and space coordinates and other light-front quantities such as momentum variables, Dirac $\gamma$ matrices, etc. Then the Dirac equation and free quark spinors are presented in this frame. In Section III we calculate the form factor of the $\pi$ meson and find that it is independent of $\omega^\mu$. In Section IV the form factors of nucleon and $\Lambda$ hyperon are calculated and found to be $\omega$-independent as well. The final Section V contains the results and conclusion.

## II. Light-Front Formalism

In ordinary space-time coordinates, a light front hyperplane is described by the invariant equation

$$\omega' \cdot x = t - \vec{n'} \cdot \vec{x} = 0 \tag{2.1}$$

where $\omega'^\mu = (1, \vec{n'})$ with $\vec{n'}$ the normal direction of the light front hyperplane. The four-vector $\omega'$ satisfies $\omega'^2 = 0$ and its direction determines the light front surface. The kinematics and dynamics constructed from the special light-front surface $\omega'^\mu = (1, 0, 0, -1)$ is the conventional light front choice. Here we develop a theory based on the general light front surface with some constant $\omega'^\mu$. In coordinate space we set up four orthogonal axes represented by the unit vectors $s'^+_\mu$, $s'^-_\mu$, $s'^1_\mu$ and $s'^2_\mu$ with

$$s'^+_\mu \equiv \omega'_\mu = (1, -\vec{n}') = (1, -n'_x, -n'_y, -n'_z) \tag{2.2$a$}$$

$$s'^-_\mu = (1, \vec{n}') = (1, n'_x, n'_y, n'_z) \tag{2.2$b$}$$

$$s'^1_\mu = (0, b_1, c_1, d_1) \tag{2.2c}$$

$$s'^2_\mu = (0, b_2, c_2, d_2) \tag{2.2d}$$

so that $\vec{s'_1}$, $\vec{s'_2}$,$\vec{n'}$ form the column vectors of a rotation matrix

$$R(\alpha, \beta, \gamma)$$

$$= \begin{pmatrix} cos\gamma cos\beta cos\alpha - sin\gamma sin\alpha & cos\gamma cos\beta sin\alpha + sin\gamma cos\alpha & -cos\gamma sin\beta \\ -sin\gamma cos\beta cos\alpha - cos\gamma sin\alpha & -sin\gamma cos\beta sin\alpha + cos\gamma cos\alpha & sin\gamma sin\beta \\ sin\beta cos\alpha & sin\beta sin\alpha & cos\beta \end{pmatrix} \tag{2.3}$$

with the Euler angles $(\alpha, \beta, \gamma)$. The four-dimensional frame spanned by the four axes is called a light-front frame. A position vector denoted by $x^\mu = (t, x, y, z)$ in coordinate space is represented in the new axes of the light-front frame as

$$x^+_{LF} = s'^+_\mu x^\mu = t - \vec{n'} \cdot \vec{x} \tag{2.4a}$$

$$x^-_{LF} = s'^-_\mu x^\mu = t + \vec{n'} \cdot \vec{x} \tag{2.4b}$$

$$x^1_{LF} = s'^1_\mu x^\mu = b_1 x + c_1 y + d_1 z \tag{2.4c}$$

$$x^2_{LF} = s'^2_\mu x^\mu = b_2 x + c_2 y + d_2 z \tag{2.4d}$$

The subscript "LF" denotes the variables of the light-front frame. From Eqs.(2.4a-2.4d), we define the inner, or scalar product of $x^\mu_{LF} = (x^+_{LF}, x^-_{LF}, x^1_{LF}, x^2_{LF})$ with itself as

$$x_{LF} \cdot x_{LF} = x_{LF\mu} x^\mu_{LF} = \frac{1}{2}(x^+_{LF} x^-_{LF} + x^-_{LF} x^+_{LF}) - x^1_{LF} x^1_{LF} - x^2_{LF} x^2_{LF}, \tag{2.5}$$

so that $x_{LF} \cdot x_{LF} = x_\mu x^\mu$ in any frame. This is equivalent to

$$(\vec{n'} \cdot \vec{x})^2 + (b_1 x + c_1 y + d_1 z)^2 + (b_2 x + c_2 y + d_2 z)^2 = x^2 + y^2 + z^2, \tag{2.6}$$

which follows from the orthogonality of the rotation matrix R in Eq.(2.3).

The axes in momentum space are then given by the row vectors $\vec{s}_1, \vec{s}_2, \vec{n}$ of the rotation matrix R in Eq.(2.3) so that, e.g.,

$$n_x = sin\beta cos\alpha \tag{2.7a}$$

$$n_y = sin\beta sin\alpha \tag{2.7b}$$

$$n_z = cos\beta. \tag{2.7c}$$

If

$$s^+_\mu \equiv \omega_\mu = (1, -\vec{n}) = (1, -n_x, -n_y, -n_z) \tag{2.2e}$$

is given, then $\alpha$ and $\beta$ are determined. The angle $\gamma$ is free and is used to fix the two axes $s^1_\mu$ and $s^2_\mu$. The four-momentum $p^\mu = (p^0, p^1, p^2, p^3)$ is obtained by projection onto the axes $s^+_\mu, s^-_\mu, s^1_\mu$ and $s^2_\mu$ of the light-front frame as

$$p^+_{LF} = s^+_\mu p^\mu = p^0 - \vec{n} \cdot \vec{p} \tag{2.8a}$$

$$P^-_{LF} = s^-_\mu p^\mu = p^0 + \vec{n} \cdot \vec{p} \tag{2.8b}$$

$$p^1_{LF} = s^1_\mu p^\mu \tag{2.8c}$$

$$p^2_{LF} = s^2_\mu p^\mu \tag{2.8d}$$

Similarly, the matrix $\gamma^\mu_{LF} = (\gamma^+_{LF}, \gamma^-_{LF}, \gamma^1_{LF}, \gamma^2_{LF})$ corresponding to the Dirac matrix $\gamma^\mu = (\gamma^0, \gamma^1, \gamma^2, \gamma^3)$ in the light-front frame is defined as

$$\gamma^+_{LF} = s^+_\mu \gamma^\mu = \gamma^0 - \vec{n} \cdot \vec{\gamma} \tag{2.9a}$$

$$\gamma^-_{LF} = s^-_\mu \gamma^\mu = \gamma^0 + \vec{n} \cdot \vec{\gamma} \tag{2.9b}$$

$$\gamma^1_{LF} = s^1_\mu \gamma^\mu \tag{2.9c}$$

$$\gamma^2_{LF} = s^2_\mu \gamma^\mu \tag{2.9d}$$

which gives $(\gamma^+_{LF})^2 = (\gamma^-_{LF})^2 = 0$, $(\gamma^1_{LF})^2 = (\gamma^2_{LF})^2 = -1$ and $\gamma^+_{LF}\gamma^-_{LF} + \gamma^-_{LF}\gamma^+_{LF} = 4$.

We define the inner product of two four-vectors $a^\mu_{LF} = (a^+_{LF},\ a^-_{LF},\ a^1_{LF},\ a^2_{LF})$ and $b^\mu_{LF} = (b^+_{LF},\ b^-_{LF},\ b^1_{LF},\ b^2_{LF})$ as

$$a_{LF} \cdot b_{LF} = \frac{1}{2}(a^+_{LF} b^-_{LF} + a^-_{LF} a^+_{LF}) - a^1_{LF} a^1_{LF} - a^2_{LF} a^2_{LF} \tag{2.10}$$

In the light-front frame, the Dirac equation becomes

$$(\gamma_{LF} \cdot p_{LF} - m) u_{LF}(p_{LF}) = 0 \tag{2.11}$$

Its spin-up $u_{LF\uparrow}(p_{LF})$ and spin-down $u_{LF\downarrow}(p_{LF})$ solutions are helicity eigenstates in the infinite-momentum frame with $p^+_{LF} \to \infty$,

$$u_{LF\uparrow}(p_{LF}) = \frac{1}{\sqrt{2mp^+_{LF}}}(p^+_{LF} + \vec{\alpha}_{LF\perp} \cdot \vec{p}_{LF\perp} + \beta m)\chi_{LF\uparrow} \tag{2.12a}$$

$$u_{LF\downarrow}(p_{LF}) = \frac{1}{\sqrt{2mp_{LF}^+}}(p_{LF}^+ + \vec{\alpha}_{LF\perp}\cdot\vec{p}_{LF\perp} + \beta m)\chi_{LF\downarrow}, \tag{2.12b}$$

in which $\vec{\alpha}_{LF\perp} = \gamma^0\vec{\gamma}_{LF\perp}$, $\beta = \gamma^0$ and

$$\chi_{LF\uparrow} = \frac{\sqrt{1-n_z}}{2}\begin{pmatrix} 1 \\ -\frac{n^R}{1-n_z} \\ 1 \\ -\frac{n^R}{1-n_z} \end{pmatrix}, \tag{2.13a}$$

$$\chi_{LF\downarrow} = \frac{\sqrt{1-n_z}}{2}\begin{pmatrix} \frac{n^L}{1-n_z} \\ 1 \\ -\frac{n^L}{1-n_z} \\ -1 \end{pmatrix} \tag{2.13b}$$

with $n^{R,L} = n_x \pm in_y$. When $n_x = n_y = 0$ and $n_z = -1$, the $\chi$'s reduce to the conventional light-cone spinors with

$$\chi_\uparrow = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \qquad \chi_\downarrow = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}. \tag{2.14}$$

The spinors satisfy the standard orthonormalization conditions

$$\bar{u}_{LF\uparrow}(p_{LF})u_{LF\uparrow}(p_{LF}) = \bar{u}_{LF\downarrow}(p_{LF})u_{LF\downarrow}(p_{LF}) = 1, \tag{2.15a}$$

$$\bar{u}_{LF\uparrow}(p_{LF})u_{LF\downarrow}(p_{LF}) = \bar{u}_{LF\downarrow}(p_{LF})u_{LF\uparrow}(p_{LF}) = 0, \tag{2.15b}$$

with

$$\chi_{LF\uparrow}^\dagger\chi_{LF\uparrow} = \chi_{LF\downarrow}^\dagger\chi_{LF\downarrow} = 1, \tag{2.16a}$$

$$\chi_{LF\uparrow}^\dagger\chi_{LF\downarrow} = \chi_{LF\downarrow}^\dagger\chi_{LF\uparrow} = 0. \tag{2.16b}$$

The relevant projection operators are

$$\Lambda_{LF+} = \frac{\gamma_{LF}^-\gamma^0}{2}, \qquad \Lambda_{LF-} = \frac{\gamma_{LF}^+\gamma^0}{2} \tag{2.17}$$

with the properties

$$\Lambda_{LF+}^2 = \Lambda_{LF+}, \qquad \Lambda_{LF-}^2 = \Lambda_{LF-}, \qquad \Lambda_{LF+}\Lambda_{LF-} = \Lambda_{LF-}\Lambda_{LF+} = 0,$$

$$\Lambda_{LF+} + \Lambda_{LF-} = 1. \tag{2.18}$$

Then $\chi_{LF\uparrow}$ and $\chi_{LF\downarrow}$ are the eigenstates of $\Lambda_{LF+}$ with the same eigenvalue 1,

$$\Lambda_{LF+}\chi_{LF\uparrow} = \chi_{LF\uparrow}, \qquad \Lambda_{LF+}\chi_{LF\downarrow} = \chi_{LF\downarrow}, \tag{2.19$a$}$$

$$\Lambda_{LF-}\chi_{LF\uparrow} = \Lambda_{LF-}\chi_{LF\downarrow} = 0. \tag{2.19$b$}$$

Some basic matrix elements calculated from these spinors are shown in Appendix 1 and Tables I, II. These matrix elements comprise $\bar{u}_{\lambda_k}\Gamma u_{\lambda_i}$ with $\Gamma = 1, \gamma^+_{LF}, \gamma^-_{LF}, \gamma^{R,L}_{LF} = \gamma^1_{LF} \pm i\gamma^2_{LF}$, $\gamma_5$, $\gamma_5\gamma^+_{LF}$, $\gamma_5\gamma^-_{LF}$, $\gamma_5\gamma^{R,L}_{LF}$, where $\lambda_k$ and $\lambda_i$ are the helicities.

We construct wave functions of hadrons in the light-front frame in the constituent quark model as follows. First, we write down the spin-isospin wave function in the hadron rest frame, then transform it into the light-front spinors in Eq.(2.14) by Melosh transformations [9]; third, rotate the wave function from the conventional light-front frame with $n_x = n_y = 0$ and $n_z = -1$ to that of Eq.(2.12) using the rotation matrix $R(\alpha, \beta, \gamma)$. These wave functions are then used to calculate electromagnetic form factors from the current $J^\mu_{LF}$ whose components are defined as

$$J^+_{LF} = s^+_\mu J^\mu, \qquad J^-_{LF} = s^-_\mu J^\mu, \qquad J^1_{LF} = s^1_\mu J^\mu, \qquad J^2_{LF} = s^2_\mu J^\mu. \tag{2.20}$$

The Dirac matrices $\gamma^\mu_{LF}$ defined by Eq.(2.9) and $\chi_{LF\uparrow,\downarrow}$ in Eq.(2.12) bring about the $\omega$(or $\vec{n}$)-dependence in the calculation of the electromagnetic current matrix elements in the following sections, where also the cumbersome notation of subscript "LF" is dropped for simplicity.

## III. $\pi$ Meson

To illustrate the calculation of the electromagnetic form factors of a spin-0 meson, we take the $\pi$ meson as an example. Let $p_1$ and $p_2$ be the momenta of the valence quark and antiquark in the meson, respectively. The $+$ and $\perp$ components of the relative momentum and total momentum are defined as

$$q_3 = x_2 p_1 - x_1 p_2, \qquad P = p_1 + p_2 \tag{3.1}$$

with Bjorken-Feynman variables $x_i (i = 1, 2)$

$$x_i = \frac{p_i^+}{P^+}, \qquad x_1 + x_2 = 1, \qquad 0 \le x_i \le 1. \tag{3.2}$$

The momentum variables of the quark and antiquark in the meson rest frame are

$$\vec{k}_{1\perp} = \vec{p}_{1\perp} - x_1 \vec{P}_\perp, \qquad \vec{k}_{2\perp} = \vec{p}_{2\perp} - x_2 \vec{P}_\perp. \tag{3.3}$$

A Gaussian momentum wave function is written in terms of the relative momentum according to the Brodsky-Huang-Lepage prescription [21],

$$\phi_o(x_i, q_3) = e^{-\frac{M_2^2}{6\alpha^2}} \tag{3.4}$$

with the hadronic size parameter $\alpha$ (and free mass operator $M_2$)

$$M_2^2 + \sum_{i=1}^{2} \frac{m_i^2}{x_i} = \sum_{i=1}^{2} \frac{\vec{k}_{i\perp}^2 + m_i^2}{x_i} = \frac{\vec{q}_{3\perp}^2 + m^2}{x_1 x_2} \tag{3.5}$$

for constituent quark mass $m_1 = m_2 = m$. The light-cone wave function of the $\pi$ meson is [6, 8]

$$\psi_\pi = N_\pi \phi_o \bar{u}_1 [P] \gamma_5 v_2, \tag{3.6}$$

where $N_\pi$ is the normalization constant determined by the charge (form factor) $F(0) = 1$ for the $\pi^+$ meson. Here $u$ is the light-cone spinor of Eq.(2.12) for the quark, $v_2 = C\bar{u}_2^T$ with the charge-conjugation operator $C$ for the antiquark, and the factor $[P] = \frac{\gamma \cdot P + m_\pi}{2m_\pi}$ with the $\pi$ meson mass $m_\pi$ originates from the Melosh rotation, while $\gamma_5$ is characteristic of the spin 0 of the meson.

In the light-front frame specified on the initial plane $\omega \cdot x = 0$, the electromagnetic current matrix elements of the free electromagnetic current for a spin-0 meson consist of the usual convection current and an explicitly $\omega$-dependent piece [14, 22],

$$< \pi(P') \mid J^\mu \mid \pi(P) >= (P + P')^\mu F(q^2) + \omega^\mu \frac{(P + P')^2}{2\omega \cdot P} g_1(q^2), \tag{3.7}$$

where $q = P' - P$, and $P'$ and $P$ are the initial and final momenta of the meson, respectively. To express $F(q^2)$ and $g_1(q^2)$ in terms of the current matrix elements, we use

$$\omega_{LF}^+ = s_\mu^+ \omega^\mu = 0, \quad \omega_{LF}^- = s_\mu^- \omega^\mu = 2, \quad \omega_{LF}^1 = s_\mu^1 \omega^\mu = 0, \quad \omega_{LF}^2 = s_{LF}^2 \omega^\mu = 0. \tag{3.8}$$

The two form factors in the Drell-Yan frame $q^+ = \omega \cdot q = 0$ can then be expressed as

$$F(q^2) = \frac{1}{2P^+} < \pi(P') \mid J^+ \mid \pi(P) >, \tag{3.9a}$$

$$g_1(q^2) = \frac{P^+}{(P+P')^2}[< \pi(P') \mid J^- \mid \pi(P) > -(P^- + P^{'-})F_1(q^2)]. \tag{3.9b}$$

In the impulse approximation the electromagnetic current matrix elements are calculated from the triangle diagram alone in the Drell-Yan frame [22] using the $\pi$-wave function of Eq.(3.6),

$$< \pi(P') \mid J^+ \mid \pi(P) >= \int d\Gamma \psi_\pi^{'\dagger} (\sum_{j=1}^{2} \bar{u}_j' e_j \frac{\gamma^+}{x_j} u_j) \psi_\pi$$

$$= N_\pi^2 \int d\Gamma \phi_o^{'*} \phi_o \{ e_1 Tr([P'][p_1'] \frac{\gamma^+}{x_1} [p_1][P][p_2]) + e_2 Tr([P'][p_1][P][p_2] \frac{\gamma^+}{x_2} [p_2']) \} , \tag{3.10}$$

where $e_i$ is the electric charge and $[p_i] = \frac{\gamma \cdot p_i + m_i}{2m_i}$ the Melosh factor with $m_i$ the mass and $p_i$ the momentum of the quark or antiquark, respectively.

## IV. Electromagnetic Form Factors for Spin-$\frac{1}{2}$ Baryons

### A. Decomposition of Electromagnetic Current Matrix Elements

If $J^\mu$ is the full electromagnetic current, the current matrix element of a baryon state consists of the standard Dirac and Pauli terms,

$$< B(P')\lambda' \mid J^\mu \mid B(P)\lambda >= \bar{u}_{\lambda'}(P')[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_2(q^2)] u_\lambda(\hat{P}) \tag{4.1}$$

with the baryon mass M.

If the free quark currents are used, the possible $\omega$-dependence of the electromagnetic current matrix elements between baryon states leads to three additional components [14] so that Eq.(4.1) becomes

$$< B(P')\lambda' \mid J^\mu \mid B(P)\lambda >= \bar{u}_{\lambda'}(P')[\alpha_1 \gamma^\mu + \alpha_2 \frac{(P'+P)^\mu}{2M} + \alpha_3 \omega^\mu$$

$$+ \alpha_4 \gamma \cdot \omega \frac{(P' + P)^\mu}{2M} + \alpha_5 \gamma \cdot \omega \omega^\mu] u_\lambda(P) \cdot \tag{4.2}$$

The quantities $\alpha_1$ and $\alpha_2$ are related to the form factors $F_1$ and $F_2$ by

$$F_1 + F_2 = \alpha_1, \tag{4.3$a$}$$

$$F_2 = -\alpha_2. \tag{4.3$b$}$$

Taking the light-front wave function of a baryon in the LCQM to calculate the matrix elements of the free current component $J^+$, we find by inspection that $< B(P')\lambda' \mid J^+ \mid B(P)\lambda >$ is linear in $P^+ + P'^+$, whereas the $\alpha_4$ term in Eq.(4.2) is proportional to $(P^+ + P'^+)^2$. Thus, $\alpha_4 = 0$ in the LCQM. Direct algebraic calculation with the use of Tables I and II in the Drell-Yan frame $q^+ = 0$ with $\vec{P}_\perp = 0$ in the initial state yields the following expressions for the form factors:

$$\alpha_1 = \frac{M}{P^+} < B(P') \uparrow \mid J^+ \mid B(P) \uparrow > -\alpha_2, \tag{4.4$a$}$$

$$\alpha_1 = \frac{M}{P^+} < B(P') \downarrow \mid J^+ \mid B(P) \downarrow > -\alpha_2, \tag{4.4$b$}$$

$$\alpha_2 = \frac{2M^2 d_1}{P^+ n^L q^L} < B(P') \uparrow \mid J^+ \mid B(P) \downarrow >, \tag{4.4$c$}$$

$$\alpha_2 = -\frac{2M^2 d_1}{P^+ n^R q^R} < B(P') \downarrow \mid J^+ \mid B(P) \uparrow >, \tag{4.4$d$}$$

$$\alpha_3 = \frac{M d_1}{n^L q^L} < B(P') \uparrow \mid J^- \mid B(P) \downarrow > -\alpha_1 \frac{M}{P^+} - \alpha_2 \frac{\vec{q}_\perp^{\,2} + 2M^2}{4MP^+}, \tag{4.4$e$}$$

$$\alpha_5 = \frac{M}{2P^+} < B(P') \uparrow \mid J^- \mid B(P) \uparrow > - \frac{M^2 d_1}{P^+ n^L q^L} < B(P') \uparrow \mid J^- \mid B(P) \downarrow > + \frac{M^2}{2(P^+)^2} \alpha_1. \tag{4.4$f$}$$

We note that the $J^-$ matrix elements in Eqs.(4.4e,f) being based on the triangle diagram alone are incomplete, but are useful nonetheless for removing the spurious $\omega$-dependent currents.

## B. Current Matrix Elements

In the light-cone formalism all quarks are on their mass shell. Relative momentum variables for the three valence quarks in a baryon may be defined [1] in terms of the quark momenta $p_i$ by

$$q_3 = (x_2 p_1 - x_1 p_2)/(x_1 + x_2) \tag{4.5a}$$

$$Q_3 = (x_1 + x_2) p_3 - x_3(p_1 + p_2), \tag{4.5b}$$

which are independent of the total momentum P. These two equations are valid for the + and $\perp$ components where $P^+ = P_0 + P_z$ and $\vec{P}_\perp = (P_x, P_y)$, etc. The Bjorken-Feynman variables $x_j$ (j=1, 2, 3) are defined as

$$x_j = \frac{p_j^+}{P^+}, \quad \sum_{j=1}^{3} x_j = 1, \quad 0 \leq x_j \leq 1. \tag{4.6}$$

The ith quark momentum variable in the baryon rest frame is given by

$$\vec{k}_{i\perp} = \vec{p}_{i\perp} - x_i \vec{P}_\perp. \tag{4.7}$$

Let us denote the spin-isospin part of the light-front wave function by $\chi_\lambda$. Then the wave function of the baryon with helicity $\lambda$ is taken to be

$$\psi_B(\lambda) = N_B \phi_o(x_3, q_3, Q_3) \chi_\lambda, \tag{4.8}$$

where $N_B$ is a normalization constant fixed by the nonzero charge (form factor at $q^2 = 0$) and the Gaussian momentum wave function is

$$\phi_o(x_i, q_3, Q_3) = e^{-\frac{M_3^2}{6\alpha^2}} \tag{4.9}$$

with light cone energy (and the free mass operator $M_3$)

$$M_3^2 + \sum_{i=1}^{3} \frac{m_i^2}{x_i} = \sum_{i=1}^{3} \frac{\vec{k}_{i\perp}^2 + m_i^2}{x_i} = -q_3^2 \frac{1 - x_3}{x_1 x_2} - \frac{Q_3^2}{x_3(1 - x_3)} + \sum_{i=1}^{3} \frac{m_i^2}{x_i}.$$

The light-front wave function is invariant under kinematic Poincaré generators. It depends only on the relative momentum variables and the longitudinal momentum fractions $x_i$ and the total mass eigenvalue, but not on $P^\mu$.

The matrix element of the free electromagnetic current in the Drell-Yan frame [23] is obtained as

$$< B\lambda' \mid J^+ \mid B\lambda >= \int d\Gamma \psi_B^{'\dagger} (\sum_{i=1}^{3} \bar{u}_i' e_i \frac{\gamma^+}{x_i} u_i) \psi_B$$

$$= N_B^2 \int d\Gamma \phi_o^*(x_i, q_3', Q_3') \phi_o(x_i, q_3, Q_3) \chi_{\lambda'}^\dagger (\sum_{i=1}^{3} \bar{u}_i' e_i \frac{\gamma^+}{x_i} u_i) \chi_\lambda. \tag{4.10}$$

In the following two subsections the wave functions of the nucleon and $\Lambda$ hyperon are employed to calculate electromagnetic form factors.

## C. Nucleon

The quark 1 and quark 2 are taken as up (down) quarks and quark 3 as a down (up) quark for the proton (neutron) case. The nucleon wave function is [1]

$$\psi_N = \pm N_N \phi_o \{ \bar{u}_2[P] \gamma_5 C \bar{u}_3^T \cdot \bar{u}_1 u_N - \bar{u}_3[P] \gamma_5 C \bar{u}_1^T \cdot \bar{u}_2 u_N \} \tag{4.11}$$

with "+" for the proton and "-" for the neutron. The normalization constant $N_N$ determined so that the Dirac form factor $F_1(0) = 1$ for the proton. It is a useful check of the symbolic and numerical codes to verify that $F_1(0) = 0$ for the neutron. Upon relabeling the quarks so that the quark 3 is always interacting with the electromagnetic field, the matrix element of the free electromagnetic current becomes

$$< p\lambda' \mid J^+ \mid p\lambda >= N_N^2 \int d\Gamma \phi_o^{'*} \phi_o \{ 2e_u \bar{u}_{\lambda'}' [p_3'] \frac{\gamma^+}{x_3} [p_3] u_\lambda \cdot Tr([P'][p_1][P][p_2])$$

$$+ (e_u + e_d) \bar{u}_{\lambda'}' [p_2] u_\lambda \cdot Tr([P'][p_3'] \frac{\gamma^+}{x_3} [p_3][P][p_1])$$

$$+ (e_u + e_d) \bar{u}_{\lambda'}' [p_1] u_\lambda \cdot Tr([P][p_3] \frac{\gamma^+}{x_3} [p_3'][P'][p_2])$$

$$+ 2e_u \bar{u}_{\lambda'}' [p_3'] \frac{\gamma^+}{x_3} [p_3][P][p_2][P'][p_1] u_\lambda$$

$$+ 2e_u \bar{u}_{\lambda'}' [p_1][P][p_2][P'][p_3'] \frac{\gamma^+}{x_3} [p_3] u_\lambda$$

$$+2e_d\bar{u}'_{\lambda'}[p_1][P][p_3]\frac{\gamma^+}{x_3}[p'_3][P'][p_2]u_\lambda\} \tag{4.12}$$

for the proton and

$$< n\lambda' \mid J^+ \mid n\lambda >=< p\lambda' \mid J^+ \mid p\lambda > (e_u \to e_d, e_d \to e_u) \tag{4.13}$$

for the neutron.

### D. $\Lambda$ Hyperon

The up-down quark pair in the $\Lambda$ hyperon is coupled to zero spin and isospin. The spin of $\Lambda$ hyperon is that of the $s$ quark. Let $u$ be quark 1, $d$ quark 2 and $s$ quark 3. The light-front wave function for the $\Lambda$ hyperon can then be written as

$$\psi_\Lambda = N_\Lambda \phi_o \bar{u}_1[P]\gamma_5 v_2 \cdot \bar{u}_3 u_\Lambda, \tag{4.14}$$

where $N_\Lambda$ is the normalization constant determined by $F_1(0) = 1$ when the charge $e_3 = -1/3$ of the $s$ quark is replaced by $+2/3$. The matrix element of the free electromagnetic current can then be written as

$$< \Lambda\lambda' \mid J^+ \mid \Lambda\lambda >= -N_\Lambda^2 \int d\Gamma \phi_o^* \phi_o \{\frac{2}{3}\bar{u}'_{\lambda'}[p_2]u_\lambda \cdot Tr([P'][p'_3]\frac{\gamma^+}{x_3}[p_3][P][p_1])$$

$$-\frac{1}{3}\bar{u}'_{\lambda'}[p_1]u_\lambda \cdot Tr([P'][p_2][P][p_3]\frac{\gamma^+}{x_3}[p'_3])$$

$$-\frac{1}{3}\bar{u}'_{\lambda'}[p'_3]\frac{\gamma^+}{x_3}[p_3]u_\lambda \cdot Tr([P'][p_1][P][p_2])\} \tag{4.15}$$

where the labels now are so that quark 3 is always interacting with the electromagnetic field.

## VI. Results and Discussion

When we construct the spin-isospin part of the wave functions of hadrons in the constituent quark model in light-front form, the quarks and antiquarks are assumed free. The Gaussian momentum wave function $\phi_0$ models the confinement interaction, is Lorentz invariant and directly linked with the

successful spectroscopy of the nonrelativistic constituent quark model. Matrix elements of free quark currents represent the lowest-order electromagnetic Feynman diagram calculation including a confinement interaction in the bound states.

The instant form is usually specified by the time $t = 0$ and the light-front plane by $x^+ = t + z$. When spin operators are taken as free, then a unitary transformation to a more general initial null plane $\omega \cdot x = 0$ becomes unavoidable. This is no problem for a Lorentz covariant theory which will not depend on the angles characterising this plane. The Poincaré group realized on the initial plane $\omega \cdot x = 0$ has seven kinematical generators and three dynamical generators. The advantages of the conventional light-front quantum field theory discussed in the introduction remain valid in a field theory constructed from the initial plane $\omega \cdot x = 0$. The Hamiltonian $P^-$ among the dynamical Poincaré generators transforms the initial plane $\omega \cdot x = 0$ to another light-front plane. Such a dynamical process evolves in the light-cone time $x^+_{LF} = t - \vec{n} \cdot \vec{x}$ of Eq.2.4b. The two transverse space coordinates $x^1_{LF}$ and $x^2_{LF}$ are not completely specified by the $\vec{n}$, but depend also on the angle $\gamma$ which is arbitrary. The most convenient choice is $\gamma = 0$. We have defined in Section II the momentum variables and Dirac matrices in terms of these coordinates. The Dirac equation (2.11) has the same form in this light-front frame.

The Dirac matrices $\gamma^\mu_{LF}$ defined by Eq.(2.9) and $\chi_{LF\uparrow,\downarrow}$ in Eq.(2.12) bring about the $\omega$-dependence in the calculation of the electromagnetic current matrix elements. In the $\pi$ meson case, the $n^R$ and $n^L$ lead to $\omega$-independent current matrix elements because $\frac{n^R n^L}{d_1^2} = 1$ . The form factor $F(q^2)$ in Eq.(3.9a) stays $\omega$-independent even if the free quark current is used in the calculations; it is physical and not influenced by the nonphysical form factor $g_1(q^2)$. The wave function of each spin-0 meson has the same structure as in Eq.(3.4) for the pion. Therefore we conclude that the physical form factor of the spin-0 meson may be extracted from the constituent quark model. The form factor calculated in the familiar light-front frame for the $\pi$ meson [6] is reliable in this sense.

For the spin-$\frac{1}{2}$ baryons the decomposition of the matrix elements lead to the additional form factor $\alpha_2$ determined by Eqs.(4.4c) or (4.4d), and $\alpha_1$ by (4.4a) or (4.4b). For the nucleon and $\Lambda$ hyperon Eqs.(4.4c) and (4.4d) give the same $\alpha_2$, Eqs. (4.4a) and (4.4b) give the same $\alpha_1$. To see this we have

developed analytical procedures to calculate the current matrix elements. For the nucleon and $\Lambda$ hyperon, we find that all $J^+$-current matrix obey $< \lambda' =\uparrow | J^+ | \lambda =\uparrow > = < \lambda' =\downarrow | J^+ | \lambda =\downarrow >$, while $< \lambda' =\uparrow | J^+ | \lambda =\downarrow > = - < \lambda' =\downarrow | J^+ | \lambda =\uparrow >$; and they are real and $\omega$-independent. Thus, Eqs.(4.4a)-(4.4d) give consistent and $\omega$-independent form factors $\alpha_2$ and $\alpha_1$. We conclude for the nucleon and the $\Lambda$ hyperon that the form factors $F_1$ and $F_2$ calculated in the constituent quark model are $\omega$-independent and not influenced by the nonphysical form factors $\alpha_3$, $\alpha_4$ and $\alpha_5$.

The free current matrix elements thus give the physical form factors in the impulse approximation without the interference of the nonphysical form factors for the spin-0 meson, the nucleon and the $\Lambda$ hyperon. We expect the N to $\Delta$ baryon transition matrix elements to have problems brought about by the spin 3/2 of the $\Delta_{3,3}$.

After this paper was submitted for publication we became aware of ref.24 where the axial-vector matrix element of the pion is found insensitive to changes of the light-cone direction in numerical studies.

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## Appendix 1

In Eq.(2.7) the Euler angles $\alpha$ and $\beta$ determine $\vec{n}$. The two axes $s^1_\mu$ and $s^2_\mu$ are fixed by the three Euler angles through the rotation matrix in Eq.(2.3). For an investigation of the $\omega$-dependence of the electromagnetic form factors, $s^1_\mu$ and $s^2_\mu$ perpendicular to $\omega_\mu$ are irrelevant. Therefore, the angle $\gamma$ is chosen to be the zero so that the $s^1_\mu$ and $s^2_\mu$ have the simple form

$$s^1_\mu = (0, -\frac{n_x n_z}{d_1}, -\frac{n_y n_z}{d_1}, d_1), \qquad s^2_\mu = (0, \frac{n_y}{d_1}, -\frac{n_x}{d_1}, 0) \qquad (A1)$$

with $d_1 = -\sqrt{1-n_z^2}$. The spin-up and spin-down spinors in Eqs. (2.12) are rewritten as

$$u_{LF\uparrow}(p_{LF}) = \frac{1}{2}\sqrt{\frac{1-n_z}{2mp^+_{LF}}} \begin{pmatrix} b_{LF} + \frac{d_1}{1-n_z} p^R_{LF} \\ -\frac{n^R}{1-n_z} b_{LF} + \frac{n^R}{d_1} p^R_{LF} \\ d_{LF} - \frac{d_1}{1-n_z} p^R_{LF} \\ -\frac{n^R}{1-n_z} d_{LF} + \frac{n^R}{d_1} p^R_{LF}, \end{pmatrix} \qquad (A1)$$

$$u_{LF\downarrow}(p_{LF}) = \frac{1}{2}\sqrt{\frac{1-n_z}{2mp^+_{LF}}} \begin{pmatrix} \frac{n^L}{1-n_z} b_{LF} - \frac{n^L}{d_1} p^L_{LF} \\ b_{LF} + \frac{d_1}{1-n_z} p^L_{LF} \\ -\frac{n^L}{1-n_z} d_{LF} + \frac{n^L}{d_1} p^L_{LF} \\ -d_{LF} - \frac{d_1}{1-n_z} p^L_{LF} \end{pmatrix} \qquad (A2)$$

with $b_{LF} = p^+_{LF} + m$ and $d_{LF} = p^+_{LF} - m$. If $n_x = n_y = 0$, $\frac{n^R}{d_1} = \frac{n^L}{d_1} = 1$ and $n_z = -1$, then the spinors reduce to the standard light-front spinors with Eq.(2.14),

$$u_\uparrow(p) = \frac{1}{2\sqrt{mp^+}} \begin{pmatrix} b \\ p_R \\ d \\ p_R \end{pmatrix}, \qquad u_\downarrow(p) = \frac{1}{2\sqrt{mp^+}} \begin{pmatrix} -p_L \\ b \\ p_L \\ -d \end{pmatrix} \qquad (A4)$$

with $b = p^+ + m$ and $d = p^+ - m$.

To calculate symbolically the matrix elements of the electromagnetic current between hadron states using REDUCE, matrix elements of $\gamma_{LF}$ for a single spinor, $\bar{u}_{LF\lambda_k}(p_k)\Gamma u_{LF\lambda_i}(p_i)$, must be calculated first. Note here that the momentum $p_k(p_i)$ may be that of a quark or a hadron. The matrix

elements are exhibited in Tables I and II with recourse to following abbreviations,

$$m_{ki} = \sqrt{x_k m_k x_i m_i}, \qquad A^{\pm}_{ki} = \frac{1}{2m_{ki}}(x_k m_i \pm x_i m_k), \qquad X_{ki} = \frac{2p^+_{LF}}{2m_{ki}} x_k x_i,$$

$$K^{R,L}_{ki} = \frac{1}{2m_{ki}}(x_k p^{R,L}_{LFi} - x_i p^{R,L}_{LFk}), \qquad W^{R,L\pm}_{ki} = \frac{2}{2m_{ki}p^+_{LF}}(m_k p^{R,L}_{LFi} \pm m_i p^{R,L}_{LFk}),$$

$$Y^{RL\pm}_{ki} = \frac{2}{2m_{ki}p^+_{LF}}(m_k m_i \pm p^R_{LFk} p^L_{LFi}), \qquad Y^{LR\pm}_{ki} = \frac{2}{2m_{ki}p^+_{LF}}(m_k m_i \pm p^L_{LFk} p^R_{LFi}).$$

The entries in Tables I,II are in agreement with those given by Konen and Weber [1], if $n_x = n_y = 0$, $n_z = -1$, and $\frac{n^R}{d_1} = \frac{n^L}{d_1} = 1$.

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**TABLE I**

| $\lambda_i$ | $\Gamma$ | $(\not{p}_{LFk})\Gamma u_{LF\lambda_i}$ | $\bar{u}_{LF\lambda_k}\Gamma u_{LF\lambda_i}$ | |
|---|---|---|---|---|
| $\uparrow$ | $1$ | $u_{LFk\uparrow}A^{+}_{ki} \;+\; u_{LFk\downarrow}\frac{n^R}{d_1}K^R_{ki}$ | $\delta_{\lambda_k,\uparrow}A^{+}_{ki} \;+\; \delta_{\lambda_k,\downarrow}\frac{n^R}{d_1}K^R_{ki}$ | |
| $\downarrow$ | $1$ | $-u_{LFk\uparrow}\frac{n^L}{d_1}K^L_{ki} \;+\; u_{LFk\downarrow}A^{+}_{ki}$ | $-\delta_{\lambda_k,\uparrow}\frac{n^L}{d_1}K^L_{ki} \;+\; \delta_{\lambda_k,\downarrow}A^{+}_{ki}$ | |
| $\uparrow$ | $\gamma^{+}_{LF}$ | $u_{LFk\uparrow}X_{ki}$ | $\delta_{\lambda_k,\uparrow}X_{ki}$ | |
| $\downarrow$ | $\gamma^{+}_{LF}$ | $u_{LFk\downarrow}X_{ki}$ | $\delta_{\lambda_k,\downarrow}X_{ki}$ | |
| $\uparrow$ | $\gamma^{-}_{LF}$ | $u_{LFk\uparrow}Y^{LR+}_{ki} \;+\; u_{LFk\downarrow}\frac{n^R}{d_1}W^{R-}_{ki}$ | $\delta_{\lambda_k,\uparrow}Y^{LR+}_{ki} \;+\; \delta_{\lambda_k,\downarrow}\frac{n^R}{d_1}W^{R-}_{ki}$ | |
| $\downarrow$ | $\gamma^{-}_{LF}$ | $-u_{LFk\uparrow}\frac{n^L}{d_1}W^{L-}_{ki} + u_{LFk\downarrow}Y^{RL+}_{ki}$ | $-\delta_{\lambda_k,\uparrow}\frac{n^L}{d_1}W^{L-}_{ki} + \delta_{\lambda_k,\downarrow}Y^{RL+}_{ki}$ | |
| $\uparrow$ | $\gamma^{R}_{LF}$ | $u_{LFk\uparrow}\frac{x_k p^R_{LFi}}{m_{ki}}$ | $\delta_{\lambda_k,\uparrow}\frac{x_k p^R_{LFi}}{m_{ki}}$ | |
| $\downarrow$ | $\gamma^{R}_{LF}$ | $2u_{LFk\uparrow}\frac{n^L}{d_1}A^{-}_{ki} \;+\; u_{LFk\downarrow}\frac{x_i p^R_{LFk}}{m_{ki}}$ | $2\delta_{\lambda_k,\uparrow}\frac{n^L}{d_1}A^{-}_{ki} \;+\; \delta_{\lambda_k,\downarrow}\frac{x_i p^R_{LFk}}{m_{ki}}$ | |
| $\uparrow$ | $\gamma^{L}_{LF}$ | $u_{LFk\uparrow}\frac{x_i p^L_{LFk}}{m_{ki}} \;-\; 2u_{LFk\downarrow}\frac{n^R}{d_1}A^{-}_{ki}$ | $\delta_{\lambda_k,\uparrow}\frac{x_i p^L_{LFk}}{m_{ki}} \;-\; 2\delta_{\lambda_k,\downarrow}\frac{n^R}{d_1}A^{-}_{ki}$ | |
| $\downarrow$ | $\gamma^{L}_{LF}$ | $u_{LFk\downarrow}\frac{x_k p^L_{LFi}}{m_{ki}}$ | $\delta_{\lambda_k,\downarrow}\frac{x_k p^L_{LFi}}{m_{ki}}$ | |

**TABLE II**

| $\lambda_i$ | $\Gamma$ | $(\not{p}_{LFk})\Gamma u_{LF\lambda_i}$ | $\bar{u}_{LF\lambda_k}\Gamma u_{LF\lambda_i}$ | |
|---|---|---|---|---|
| $\uparrow$ | $\gamma_5$ | $-u_{LFk\uparrow}A^-_{ki} \ + \ u_{LFk\downarrow}\frac{n^R}{d_1}K^R_{ki}$ | $-\delta_{\lambda_k,\uparrow}A^-_{ki} \ + \ \delta_{\lambda_k,\downarrow}\frac{n^R}{d_1}K^R_{ki}$ | |
| $\downarrow$ | $\gamma_5$ | $u_{LFk\uparrow}\frac{n^L}{d_1}K^L_{ki} \ + \ u_{LFk\downarrow}A^-_{ki}$ | $\delta_{\lambda_k,\uparrow}\frac{n^L}{d_1}K^L_{ki} \ + \ \delta_{\lambda_k,\downarrow}A^-_{ki}$ | |
| $\uparrow$ | $\gamma_5\gamma^+_{LF}$ | $-u_{LFk\uparrow}X_{ki}$ | $-\delta_{\lambda_k,\uparrow}X_{ki}$ | |
| $\downarrow$ | $\gamma_5\gamma^+_{LF}$ | $u_{LFk\downarrow}X_{ki}$ | $\delta_{\lambda_k,\downarrow}X_{ki}$ | |
| $\uparrow$ | $\gamma_5\gamma^-_{LF}$ | $u_{LFk\uparrow}Y^{LR-}_{ki} \ - \ u_{LFk\downarrow}\frac{n^R}{d_1}W^{R+}_{ki}$ | $\delta_{\lambda_k,\uparrow}Y^{LR-}_{ki} \ - \ \delta_{\lambda_k,\downarrow}\frac{n^R}{d_1}W^{R+}_{ki}$ | |
| $\downarrow$ | $\gamma_5\gamma^-_{LF}$ | $-u_{LFk\uparrow}\frac{n^L}{d_1}W^{L+}_{ki} - u_{LFk\downarrow}Y^{RL-}_{ki}$ | $-\delta_{\lambda_k,\uparrow}\frac{n^L}{d_1}W^{L+}_{ki} - \delta_{\lambda_k,\downarrow}Y^{RL-}_{ki}$ | |
| $\uparrow$ | $\gamma_5\gamma^R_{LF}$ | $-u_{LFk\uparrow}\frac{x_k p^R_{LFi}}{m_{ki}}$ | $-\delta_{\lambda_k,\uparrow}\frac{x_k p^R_{LFi}}{m_{ki}}$ | |
| $\downarrow$ | $\gamma_5\gamma^R_{LF}$ | $-2u_{LFk\uparrow}\frac{n^L}{d_1}A^+_{ki} + u_{LFk\downarrow}\frac{x_i p^R_{LFk}}{m_{ki}}$ | $-2\delta_{\lambda_k,\uparrow}\frac{n^L}{d_1}A^+_{ki} + \delta_{\lambda_k,\downarrow}\frac{x_i p^R_{LFk}}{m_{ki}}$ | |
| $\uparrow$ | $\gamma_5\gamma^L_{LF}$ | $-u_{LFk\uparrow}\frac{x_i p^L_{LFk}}{m_{ki}} - 2u_{LFk\downarrow}\frac{n^R}{d_1}A^+_{ki}$ | $-\delta_{\lambda_k,\uparrow}\frac{x_i p^L_{LFk}}{m_{ki}} - 2\delta_{\lambda_k,\downarrow}\frac{n^R}{d_1}A^+_{ki}$ | |
| $\downarrow$ | $\gamma_5\gamma^L_{LF}$ | $u_{LFk\downarrow}\frac{x_k p^L_{LFi}}{m_{ki}}$ | $\delta_{\lambda_k,\downarrow}\frac{x_k p^L_{LFi}}{m_{ki}}$ | |